

Binary Pulsar Constraints on the Parameterized post-Einsteinian Framework

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We constrain the parameterized post-Einsteinian framework with binary pulsar observations of orbital period decay due to gravitational wave emission. This framework proposes to enhance the amplitude and phase of gravitational waveform templates through post-Einsteinian parameters to search for generic deviations from General Relativity in gravitational wave data. Such enhancements interpolate between General Relativity and alternative theory predictions, but their magnitude must be such as to satisfy all current experiments and observations. The data that currently constrains the parameterized post-Einsteinian framework the most is the orbital period decay of binary pulsars. We use such observations to place upper limits on the magnitude of post-Einsteinian parameters, which will be critical when gravitational waves are detected and this framework is implemented.

Introduction. Gravitational waves (GWs) will allow us to learn about the gravitational interaction in regimes that are currently inaccessible by more conventional, electromagnetic means. Binary black hole and neutron star mergers, for example, lead to gravitational fields that are intensely strong and highly dynamical, a regime where General Relativity (GR) has not yet been tested. GW theorists and data analysts will need to be able to make quantitative statements about the confidence that a certain event is not just a GW detection but one consistent with GR.

The parameterized post-Einsteinian (ppE) framework [1] was devised precisely for this purpose: to search for statistically significant GR deviations or *anomalies* in GW data and, in their absence, to quantify the degree of belief that a GW event is purely described by GR. This framework enhances the waveform templates used in matched-filtering through parameters that characterize GR deformations. In practice, this is achieved by adding to GR templates amplitude and phase correction, with magnitudes depending on certain ppE parameters.

Any framework that modifies GR must comply with Solar System and binary pulsar observations. These measurements already strongly constrain GR deviations in weak and moderately strong fields. The ppE framework was constructed on a maxim of compliance with current observations, which can be enforced by requiring that the magnitude of the ppE correction be such as to satisfy current constraints. Until now, this maxim had not been quantitatively enforced because it was thought that it would be difficult to relate the ppE deformations to Solar System or binary pulsar observations.

We have here found a relatively simple way to relate the ppE framework to current experiments. As shown in [1], modifications to the dissipative and conservative sectors of the theory lead structurally to similar ppE corrections to the waveform. We find that to constrain the ppE framework with current experiments at least initially, it suffices to consider dissipative corrections only, while keeping the conservative sector unmodified. Such dissipative corrections modify the amount of orbital binding energy carried away by GWs, which affects directly

the orbital period decay in binary pulsars.

The relatively recent discovery of the binary pulsar, PSR J0737-3039 [2], has provided particularly powerful GR tests [3]. This pulsar is highly relativistic, with an orbital period of about 2 hours, and has an orbital geometry favorable for measuring quantities such as the Shapiro delay with sub-percent precision. Such data has been recently used to constrain alternative theories of gravity to new levels [4].

In this paper, we relate such sub-percent accurate measurements of the orbital decay of PSR J0737-3039 to constrain the ppE framework and its templates. Because of the structure of the ppE correction to GWs, these constraints are *relational*, i.e., they are of the form $|\gamma|f^c \leq F(\delta, \vec{\lambda})$, where (γ, c) are ppE parameters, f is the GW frequency and $F(\delta, \vec{\lambda})$ is some function of the accuracy δ to which the orbital decay has been measured and system parameters $\vec{\lambda}$, such as the mass ratio and total mass of the binary. Thus, given a value for c , the magnitude of γ is constrained by binary pulsar observations to be less than some number related to δ , f and $\vec{\lambda}$. The relational constraint found in this paper will be crucial in the implementation of the ppE framework in a realistic data analysis pipeline once GWs are detected. In the rest of this paper, we follow mostly the conventions of [5] with geometric units $G = c = 1$.

Basics of the ppE Framework. The main GW observable is the so-called response function, which describes how an interferometer reacts to an impinging GW. In GR, this function is given by

$$h_{\text{GR}}(t) \equiv F_+ h_+^{\text{GR}}(t) + F_\times h_\times^{\text{GR}}(t), \quad (1)$$

where $F_{+,\times}$ are beam-pattern functions and $h_{+,\times}^{\text{GR}}$ are the plus and cross GW polarizations, built from contractions of the metric perturbation with certain polarization tensors [5]. For quasi-circular binaries, these polarizations can be Fourier transformed in the stationary-phase ap-

proximation [6–8] to yield

$$\begin{aligned}\tilde{h}_+^{\text{GR}} &= -\frac{\mathcal{M}}{D_L} \frac{u^{2/3}}{\sqrt{2\dot{F}}} (1 + \cos^2 \iota) e^{-i(\Psi_{\text{GR}} + \pi/4 - 2\beta)}, \\ \tilde{h}_\times^{\text{GR}} &= -\frac{\mathcal{M}}{D_L} \frac{u^{2/3}}{\sqrt{2\dot{F}}} (2 \cos \iota) e^{-i(\Psi_{\text{GR}} - \pi/4 - 2\beta)},\end{aligned}\quad (2)$$

where (ι, β) are the inclination and polarization angles, D_L is the luminosity distance from source to observer and \dot{F} is the rate of change of the orbital frequency due to GW emission. This frequency is defined as $F \equiv (1/2\pi)\dot{\Psi}$, where Ψ is the orbital phase, and it is also equal to half the Fourier or GW frequency f , i.e., $F = f/2$. The quantity Ψ_{GR} is the GR GW phase in the Fourier domain, which can be computed via

$$\Psi_{\text{GR}}(f) = 2\pi \int^{f/2} \frac{F'}{\dot{F}} \left(2 - \frac{f}{F'}\right) dF'. \quad (3)$$

The quantity $u \equiv \pi \mathcal{M} f$ is a dimensionless frequency parameter, where $\mathcal{M} = \eta^{3/5} m$ is the chirp mass, with $\eta = m_1 m_2 / m^2$ the symmetric mass ratio and $m = m_1 + m_2$ the total mass. From Eq. (1), it follows that the Fourier transform of the response function in the stationary phase approximation is simply $\tilde{h}_{\text{GR}} = F_+ \tilde{h}_+^{\text{GR}} + F_\times \tilde{h}_\times^{\text{GR}}$.

The ppE framework proposes that one enhances the GR response function via an amplitude and a phase correction. In the Fourier domain and in the stationary phase approximation, one can parameterize the response function for a GW from an unequal-mass, binary, quasi-circular inspiral as [1, 9]

$$\tilde{h} = \tilde{h}_{\text{GR}} (1 + \alpha \eta^c u^a) e^{i\beta \eta^d u^b}, \quad (4)$$

where (α, c, a) are ppE amplitude parameters and (β, d, b) are ppE phase parameters. Such a correction arises generically if one modifies $\dot{F} = \dot{E} (dE_b/dF)^{-1}$, which in turn can arise either due to a modification to the GW luminosity \dot{E} (the dissipative sector) or to the orbital binding energy E_b (the conservative sector). As explained in [1], this degeneracy breaks the one-to-one mapping from a ppE waveform modification to a specific alternative theory, as one cannot tell whether the change arose in the dissipative or conservative sector.

Gravitational Wave Luminosity. We now compute the energy carried by ppE GWs. As is clear from Eq. (2), the GW amplitude depends on \dot{F} , which by the chain rule can be related to \dot{E} as explained below Eq. (4). We can construct \dot{E} directly from h_+ or h_\times via

$$\dot{E} = \frac{\pi}{2} f^2 D_L^2 \dot{f}_{\text{GR}} \int d\Omega \left(|\tilde{h}_+|^2 + |\tilde{h}_\times|^2 \right), \quad (5)$$

where \dot{f}_{GR} is the rate of change of the GW frequency, and $d\Omega = \sin \iota d\iota d\beta$ integrates over the (ι, β) dependence of the waveform. Notice that Eq. (5) agrees with Eq. (2.38) in [6]. Substituting for \tilde{h} using Eq. (4), we find

$$\dot{E} = \dot{E}_{\text{GR}} |1 + \alpha \eta^c u^a|^2, \quad (6)$$

where \dot{E}_{GR} is the GR expectation for the GW luminosity:

$$\dot{E}_{\text{GR}} = \frac{\pi}{2} \dot{f}_{\text{GR}} f^2 D_L^2 \int d\Omega \left(|\tilde{h}_+^{\text{GR}}|^2 + |\tilde{h}_\times^{\text{GR}}|^2 \right), \quad (7)$$

One can also obtain an expression for the GW luminosity in terms of the GW phase only, as this also depends on \dot{F} as shown in Eq. (3). Noting that $d^2\Psi/df^2 = \pi \dot{F}^{-1}$, we can write the GW luminosity as

$$\dot{E} = -\frac{1}{6} \dot{f}_{\text{GR}}^2 \mathcal{M}^2 u^{-1/3} \frac{d^2\Psi}{df^2}. \quad (8)$$

Since $\Psi = \Psi_{\text{GR}} + \beta \eta^d u^b$, we find that

$$\dot{E} = \dot{E}_{\text{GR}} \left[1 + \pi^2 \mathcal{M}^2 \beta \eta^d b (b-1) u^{b-2} \left(\frac{d^2\Psi_{\text{GR}}}{df^2} \right)^{-1} \right], \quad (9)$$

where \dot{E}_{GR} can be written in terms of the GW phase as

$$\dot{E}_{\text{GR}} = -\frac{1}{6} \dot{f}_{\text{GR}}^2 \mathcal{M}^2 u^{-1/3} \frac{d^2\Psi_{\text{GR}}}{df^2}. \quad (10)$$

This shows that measurement of \dot{E} from a *circular* binary pulsar would allow us to constrain both the amplitude and the phase ppE parameters. All known binary pulsars, however, are in eccentric orbits. The \dot{f}_{GR} used in Eqs. (5) and (8), or equivalently the \dot{E}_{GR} used in Eqs. (6) and (9), must be that of an eccentric orbit, namely [10]

$$\dot{E}_{\text{GR}} = -\frac{32}{5} \eta^2 \frac{m^5}{a^5} (1 - e^2)^{-7/2} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right). \quad (11)$$

The ppE corrections to \dot{E} computed in Eqs. (6) and (9) are built from circular, non-spinning ppE templates, the only ones currently available. The ppE corrections to the GW luminosity computed here are really only the leading order terms in a post-circular expansion [8], i.e., an expansion in small eccentricity e . If the eccentricity is small, such as with PSR J0737-3039 ($e = 0.088$), such an approximation is well-justified and will not strongly affect the constraints we place on the ppE framework.

Orbital Period Decay. The GW luminosity enters into binary pulsar observables through the orbital decay: $\dot{P}/P = (3/2) \dot{E}_b/E_b = -(3/2) \dot{E}/E_b$, where in the second equality we used energy balance: the amount of binding energy lost by the system is equal to minus the amount of energy carried away by GWs, $\dot{E}_b = -\dot{E}$. Using Eq. (6) and (9), we then find that the \dot{P} corrected by amplitude ppE parameters is

$$\frac{\dot{P}}{P} = \left(\frac{\dot{P}}{P} \right)_{\text{GR}} (1 + 2\alpha \eta^c u^a), \quad (12)$$

while that corrected by phase ppE parameters is

$$\frac{\dot{P}}{P} = \left(\frac{\dot{P}}{P} \right)_{\text{GR}} \left(1 + \frac{48}{5} \beta \eta^d b (b-1) u^{b+5/3} \right). \quad (13)$$

The quantity $(\dot{P}/P)_{\text{GR}}$ stands for the orbital decay in GR for an eccentric inspiral, namely [10]

$$\left(\frac{\dot{P}}{P}\right)_{\text{GR}} = -\frac{96}{5} \frac{\eta m^3}{a^4} (1-e^2)^{-7/2} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right). \quad (14)$$

Recall again that the ppE corrections [the second terms inside the parenthesis of Eqs. (12) and (13)], are only valid to leading order in the post-circular approximation. In deriving these expressions, we have used the fact that the observed \dot{P}/P is very close to the GR value: $(\dot{P}/P)_{\text{obs}} = (\dot{P}/P)_{\text{GR}}(1 + \delta)$. The observational error $\delta \equiv (\delta\dot{P})/\dot{P} \ll 1$, meaning that the error on \dot{P} dominates over the error on P .

Since binary pulsar observations have confirmed GR up to observational error, we can now place relation constraints on the ppE framework. Focusing first on the amplitude ppE parameters, we find that

$$|\alpha| \leq \frac{1}{2} \frac{\delta}{\eta^c u^a}. \quad (15)$$

For the phase ppE parameter,

$$|\beta| \leq \frac{5}{48|b|} \frac{\delta}{|b-1| \eta^d u^{b+5/3}}. \quad (16)$$

A binary pulsar measurement of \dot{P} to an accuracy δ allows us to constrain α and β , given some value for (a, b, c, d) , the symmetric mass ratio and the GW frequency, or equivalently, the orbital period.

Before proceeding, let us first discuss the apparent degeneracy between the amplitude and the phase correction. Comparing Eqs. (12) and (13), one realizes that if changes to the GW amplitude and phase are due to the *same* mechanism (for example, if only \dot{E} is modified), then we must have $a = b + 5/3$, $c = d$, and $\beta = 5\alpha/[48b(b-1)]$. The ppE scheme, however, allows for modifications to *both* the dissipative (\dot{E}) sector and the conservative (E_b) sector. If both sectors are modified, there will be two sets of independent modifications, one to the phase and one to the amplitude. If a ppE correction is introduced to the GW amplitude, then it is constrained by Eq. (12); if a ppE correction is introduced to the GW phase, then it is constrained by Eq. (13). These constraints on the amplitude and phase ppE parameters are thus *independent* from each other, even though a constraint or measurement of them would not allow a one-to-one mapping to a conservative or dissipative modification. Thus, conservative and dissipative modifications are in fact degenerate, even though the phase and amplitude measurements are not.

Binary Pulsar Constraint. Let us now employ the recent measurements of [2, 3] on PSR J0737-3039 to constrain (α, β) . This binary consists of two neutron stars with component masses $m_1 = 1.3381(7) M_\odot$ and $m_2 = 1.2489(7) M_\odot$ in an almost circular orbit with eccentricity $e = 0.0877775(9)$ and period $P = 8834.535000(4)$ s.

The symmetric mass ratio is $\eta \simeq 0.24970$, the chirp mass is $\mathcal{M} \simeq 5.5399 \times 10^{-6}$ s, the GW frequency is $f = 2/P \simeq 2.263842976 \times 10^{-4}$ Hz, and the reduced frequency is $u \simeq 3.940046595 \times 10^{-9}$. The time derivative of the period is measured to be $\dot{P} = -1.252(17) \times 10^{-12}$, which implies an uncertainty of $\delta = 0.017 \times 10^{-12} / (1.252 \times 10^{-12}) \simeq 10^{-2}$. This uncertainty is comparable to the systematic error in the ppE parameters due to the neglect of eccentricity effects in PSR J0737-3039, which roughly scale as $e^2 \simeq 0.0077$. An increase in the accuracy of the \dot{P} measurement, reducing δ , would not allow us to place stronger constraints until the ppE templates are extended to include eccentricity.

Figure 1 plots the double binary pulsar constraints on $(|\alpha|, |\beta|)$ as a function of the exponent ppE parameters (a, b) for fixed (c, d) . The area above the curves is excluded by binary pulsar observations, forcing (α, β) to be smaller than a value which depends on (a, b, c, d) . Generally, if $a < -0.4$ then $|\alpha| < 10^{-6}$ for all plotted values of c , while if $b < -1.9$ then $|\beta| < 10^{-6}$ for all plotted values of d . For $a > 0.2$ and $b > -4/3$, α and β can be greater than unity for all plotted values of (c, d) . This makes sense: as (a, b) become large and positive, the ppE correction becomes smaller for low reduced frequency sources.

These constraints are consistent with other constraints on GR deviations from binary pulsars. For example, one can place a generic constraint on the time-variation of Newton's constant G with a binary pulsar observation [11, 12]: $\dot{G}/G \leq (\delta P)/(2P)$, where δP is whatever part of \dot{P} that is otherwise unexplained. Using PSR J0737-3039 [2, 3] one infers that $\dot{G}/G < 3 \times 10^{-11} \text{ yr}^{-1}$. Allowing for Newton's constant to be a linear function of time leads to a modification that can be mapped to Eq. (4) with $|\alpha| = (5/512)(\dot{G}/G)M$, $c = 3/5$ and $a = -8/3$ for the amplitude parameters, and $|\beta| = (25/65536)(\dot{G}/G)M$, $d = 3/5$ and $b = -13/3$ for the phase parameters [13]. From the binary pulsar constraint on \dot{G}/G , we then infer that $|\alpha| \lesssim 10^{-25}$ and $|\beta| \lesssim 10^{-27}$, which is consistent with Eqs. (15) and (16) and Fig. 1.

Our constraints on α look extremely strong (e.g., for $a < -2$, then $|\alpha| \lesssim 10^{-20}$). However, this does not imply that the unconstrained region (below the curves in Fig. 1) is uninteresting. For example, constraining \dot{G}/G below 10^{-12} yr^{-1} or 10^{-13} yr^{-1} implies constraining $|\alpha|$ below 10^{-25} . This is interesting as there are GR modifications that suggest \dot{G}/G deviations of this order may be present [14]. On the other hand, the smallness of the y-axis of Fig. 1 does suggest that α and β are perhaps not the best “coordinates” with which to measure GR deviations when a and b are sufficiently negative.

We conclude this discussion with some caveats on the constraints we find. First, we have here neglected the effect of eccentricity in the ppE correction, although this is accounted for in the GR part of \dot{P}/P . We have not studied eccentric ppE templates because these do not yet exist. This is due to the difficulty in constructing analytically simple Fourier transforms of eccentric inspiral waveforms in the stationary phase approximation [8]. Second,

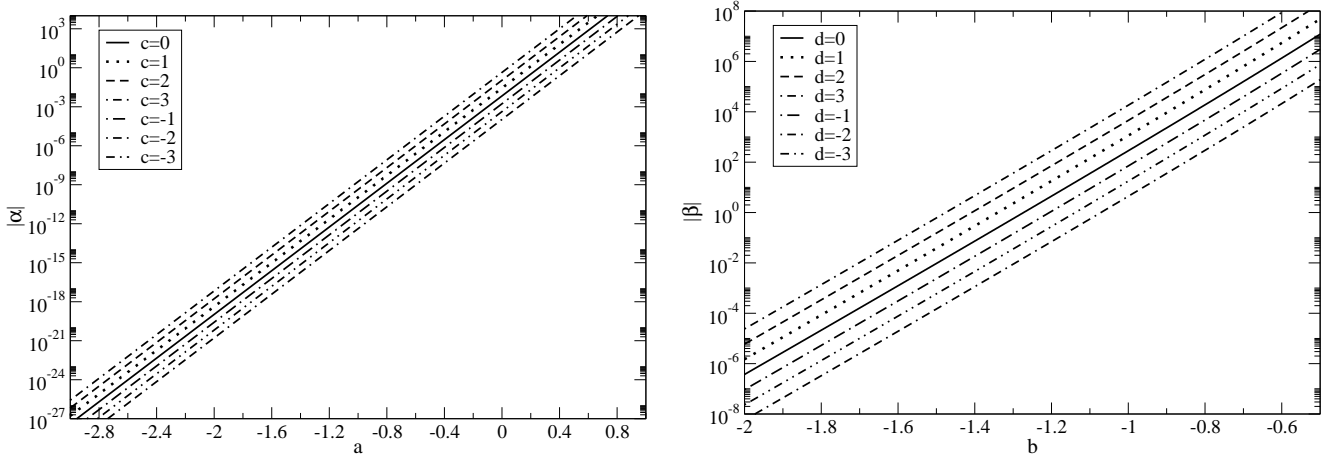


FIG. 1: Left: Constraint on $|\alpha|$ as a function of a for fixed c . Right: Constraint on $|\beta|$ as a function of b for fixed d . The area below the curves is allowed, while the area above is ruled out.

because of the ppE degeneracy between conservative and dissipative corrections, we have only examined dissipative ones here; recall, however, that amplitude and phase measurements are truly independent. Since the conservative sector can be thought of as unmodified, this allows us to use the GR measured values for the components' masses, as the Shapiro time delay and periastron precession are the same as in GR (leading to identical results for m_1 and m_2 in GR and in the ppE extended theory). We could have instead allowed for both conservative and dissipative corrections, and then analyzed how these affect *all* binary pulsar observables. A combined analysis of all these effects will presumably lead to a stronger bound on the ppE parameters; we leave this to future work. We note that we could have studied constraints on ppE from Solar System observations. However, the exquisite accuracy of the double binary pulsar measurements, and the fact that this is a much stronger-field source than any Solar System one, means that Solar System constraints will not be as stringent as the ones discussed here.

Implications for GW Data Analysis. Once GWs are detected, one would like to implement the ppE framework in a realistic data analysis pipeline. Such a pipeline will likely employ techniques from Bayesian analysis [15], which relies heavily on the priors chosen for the parameters searched over. The prior tells us whether certain regions of parameter space are allowed or likely to occur in Nature. The priors for the ppE parameters should be constructed following current Solar System and binary pulsar constraints. Equations (15) and (16) represent the most stringent prior found to date for these parameters using binary pulsar observations.

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